

## Quick Notes on the Effect of Treatment on the Treated (ETT)

Consider the effect of treatment on the treated, where  $A$  is a binary treatment,  $Y$  is outcome, and  $C$  is all confounders. Then, on the additive scale, this quantity is defined as  $E[Y_{a=1} - Y_{a=0}|A = 1] = \theta - \psi$  where  $\theta = E[Y_{a=1}|A = 1]$  and  $\Psi = E[Y_{a=0}|A = 1]$ . Then make the following assumptions:

1. Consistency:  $Y = Y_A$  almost surely;
2. No unmeasured confounding:  $A \perp Y_a|C$ ;
3. Positivity:  $\frac{P(A=0|C)}{P(A=1|C)} > 0$  almost surely.

In the absence of missing data, it is well known that under assumption 1,  $\theta = E[Y|A = 1]$ .

Then,

$$\begin{aligned}\psi &= E[Y_{a=0}|A = 1] \\ &= \frac{1}{P(A = 1)} E \left[ (1 - A) \frac{P(A = 1|C)}{P(A = 0|C)} Y \right]\end{aligned}$$

Reference: Angrist, J.D. and Pischke, J.S. (2009). *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton University Press.